

## CP VIOLATION IN THE B SYSTEM\*

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## ABSTRACT

Strategies for extracting CKM phases from non-leptonic  $B$  decays are reviewed briefly. Both general aspects and some recent developments including CP-violating asymmetries, the  $B_s$  system in light of a possible width difference  $\Delta\Gamma_s$ , and triangle relations among  $B$  decay amplitudes are discussed. The role of electroweak penguins in these strategies is illustrated briefly.

## 1. Setting the Scene

Within the Standard Model of electroweak interactions<sup>1</sup>, CP violation is closely related to the Cabibbo–Kobayashi–Maskawa matrix<sup>2,3</sup> (CKM matrix) connecting the electroweak eigenstates of the  $d$ -,  $s$ - and  $b$ -quarks with their corresponding mass eigenstates:

$$\begin{pmatrix} d \\ s \\ b \end{pmatrix}_{\text{EW}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \cdot \begin{pmatrix} d \\ s \\ b \end{pmatrix}_{\text{mass}}. \quad (1)$$

Whereas a single real parameter – the Cabibbo angle  $\Theta_C$  – suffices to parametrize the CKM matrix in the case of two fermion generations<sup>2</sup>, three generalized Cabibbo-type angles and a single *complex phase* are needed in the three generation case<sup>3</sup>. This complex phase is the origin of CP violation within the Standard Model.

The  $B$ -meson system is expected to provide a very fertile ground for testing the Standard Model description of CP violation<sup>4</sup>. Concerning such tests, the central target is the usual “non-squashed” unitarity triangle<sup>5</sup> of the CKM matrix which is a graphical illustration of the fact that the CKM matrix is unitary. At present the unitarity triangle can only be constrained indirectly<sup>6,7</sup> through experimental data from CP-violating effects in the neutral  $K$  meson system,  $B^0$ – $\bar{B}^0$  mixing and from certain tree decays measuring  $|V_{cb}|$  and  $|V_{ub}|/|V_{cb}|$ . It should, however, be possible to determine independently the three angles  $\alpha$ ,  $\beta$  and  $\gamma$  of the unitarity triangle directly at future  $B$  physics facilities<sup>8</sup> by measuring CP-violating effects in  $B$  decays<sup>4</sup>. Obviously one of the most exciting questions related to these measurements is whether the results for  $\alpha$ ,  $\beta$ ,  $\gamma$  will agree one day or will not. The latter possibility would signal “new” physics<sup>9</sup> beyond the Standard Model.

As far as CP-violating phenomena in the  $B$  system and strategies for extracting CKM phases are concerned, the key role is played by *non-leptonic*  $B$  decays. In order to analyze such transitions theoretically, one uses low energy effective Hamiltonians that are calculated by making use of the *operator product expansion* yielding transition matrix elements of the structure

$$\langle f | \mathcal{H}_{\text{eff}} | i \rangle = \sum_k C_k(\mu) \langle f | Q_k(\mu) | i \rangle. \quad (2)$$

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The operator product expansion allows one to separate the short-distance contributions to Eq. (2) from the long-distance contributions which are described by perturbative Wilson coefficient functions  $C_k(\mu)$  and non-perturbative hadronic matrix elements  $\langle f|Q_k(\mu)|i\rangle$ , respectively. As usual,  $\mu$  denotes an appropriate renormalization scale.

In the case of  $|\Delta B| = 1$ ,  $\Delta C = \Delta U = 0$  transitions relevant for the following discussion we have

$$\mathcal{H}_{\text{eff}} = \mathcal{H}_{\text{eff}}(\Delta B = -1) + \mathcal{H}_{\text{eff}}(\Delta B = -1)^\dagger \quad (3)$$

with

$$\mathcal{H}_{\text{eff}}(\Delta B = -1) = \frac{G_F}{\sqrt{2}} \left[ \sum_{j=u,c} V_{jq}^* V_{jb} \left\{ \sum_{k=1}^2 Q_k^{jq} C_k(\mu) + \sum_{k=3}^{10} Q_k^q C_k(\mu) \right\} \right]. \quad (4)$$

Here  $\mu = \mathcal{O}(m_b)$ ,  $Q_k^{jq}$  are four-quark operators and the label  $q \in \{d, s\}$  corresponds to  $b \rightarrow d$  and  $b \rightarrow s$  transitions. The index  $k$  distinguishes between “current-current” ( $k \in \{1, 2\}$ ), QCD ( $k \in \{3, \dots, 6\}$ ) and electroweak ( $k \in \{7, \dots, 10\}$ ) “penguin” operators that are related to tree-level, QCD and electroweak penguin processes, respectively. The evaluation of such low energy effective Hamiltonians beyond the leading logarithmic approximation has been reviewed recently in Ref.<sup>7</sup> and the reader is referred to that paper for the technicalities. There also the four-quark operators are given explicitly and numerical values for the Wilson coefficient functions can be found. Beyond the leading logarithmic approximation problems arise from renormalization scheme dependences which require to include certain quark-level matrix elements at  $\mu = \mathcal{O}(m_b)$  in order to have a consistent calculation<sup>10</sup>.

## 2. Strategies for Extracting CKM Phases

In this section strategies for extracting CKM phases are reviewed briefly. I will discuss CP-violating asymmetries, the  $B_s$  system in light of a possible width difference  $\Delta\Gamma_s$  between the  $B_s$  mass eigenstates, and triangle relations among certain non-leptonic  $B$  decay amplitudes.

### 2.1. CP-violating Asymmetries in $B_d$ Decays

A particular simple and interesting situation arises if we restrict ourselves to  $B_d$  decays into CP self-conjugate final states  $|f\rangle$  satisfying

$$(\mathcal{CP})|f\rangle = \pm|f\rangle. \quad (5)$$

In that case the corresponding time-dependent CP asymmetry can be expressed as

$$a_{\text{CP}}(t) \equiv \frac{\Gamma(B_d^0(t) \rightarrow f) - \Gamma(\overline{B}_d^0(t) \rightarrow f)}{\Gamma(B_d^0(t) \rightarrow f) + \Gamma(\overline{B}_d^0(t) \rightarrow f)} = \mathcal{A}_{\text{CP}}^{\text{dir}}(B_d \rightarrow f) \cos(\Delta M_d t) + \mathcal{A}_{\text{CP}}^{\text{mix-ind}}(B_d \rightarrow f) \sin(\Delta M_d t), \quad (6)$$

where we have separated the *direct* CP-violating contributions from the *mixing-induced* CP-violating contributions which are characterized by

$$\mathcal{A}_{\text{CP}}^{\text{dir}}(B_d \rightarrow f) \equiv \frac{1 - |\xi_f^{(d)}|^2}{1 + |\xi_f^{(d)}|^2} \quad (7)$$

and

$$\mathcal{A}_{\text{CP}}^{\text{mix-ind}}(B_d \rightarrow f) \equiv \frac{2 \text{Im} \xi_f^{(d)}}{1 + |\xi_f^{(d)}|^2}, \quad (8)$$

respectively. Here direct CP violation refers to CP-violating effects arising directly in the corresponding decay amplitudes whereas mixing-induced CP violation is related to interference effects between  $B_d^0$ - $\overline{B}_d^0$  mixing and decay processes. The time-dependent rates  $\Gamma(B_d^0(t) \rightarrow f)$  and  $\Gamma(\overline{B}_d^0(t) \rightarrow f)$  in Eq. (6) describe the time-evolutions due to  $B_d^0$ - $\overline{B}_d^0$  oscillations for initially, i.e. at  $t = 0$ , present  $B_d^0$  and  $\overline{B}_d^0$  mesons, respectively, and  $\Delta M_d$  denotes the mass difference of the  $B_d$  mass eigenstates. In general the observable

$$\xi_f^{(d)} \equiv \mp \exp(-i 2\beta) \frac{\sum_{j=u,c} V_{jq}^* V_{jb} \langle f | \mathcal{Q}^{jq} | \overline{B}_d^0 \rangle}{\sum_{j=u,c} V_{jq} V_{jb}^* \langle f | \mathcal{Q}^{jq} | B_d^0 \rangle}, \quad (9)$$

where  $2\beta$  is related to the weak  $B_d^0$ - $\overline{B}_d^0$  mixing phase and  $\mathcal{Q}^{jq}$  denotes the combination of four-quark operators and Wilson coefficients appearing in Eq. (4), suffers from large hadronic uncertainties that are introduced through the hadronic matrix elements in Eq. (9). There is, however, a very important special case where these uncertainties cancel. It is given if  $B_d \rightarrow f$  is dominated by a single CKM amplitude. In that case  $\xi_f^{(d)}$  takes the simple form

$$\xi_f^{(d)} = \mp \exp[-i(2\beta - \phi_D^{(f)})], \quad (10)$$

where  $\phi_D^{(f)}$  is a characteristic decay phase that is given by

$$\phi_D^{(f)} = \begin{cases} -2\gamma & \text{for dominant } \bar{b} \rightarrow \bar{u}u\bar{q} \text{ CKM amplitudes } (q \in \{d, s\}) \text{ in } B_d \rightarrow f \\ 0 & \text{for dominant } \bar{b} \rightarrow \bar{c}c\bar{q} \text{ CKM amplitudes } (q \in \{d, s\}) \text{ in } B_d \rightarrow f. \end{cases} \quad (11)$$

Applications and well-known examples of this formalism are the decays  $B_d \rightarrow J/\psi K_S$  and  $B_d \rightarrow \pi^+\pi^-$ . If one goes through the relevant Feynman diagrams contributing to the former channel one finds that it is dominated to excellent accuracy by the  $\bar{b} \rightarrow \bar{c}c\bar{s}$  CKM amplitude. Therefore the decay phase vanishes and we have

$$\mathcal{A}_{\text{CP}}^{\text{mix-ind}}(B_d \rightarrow J/\psi K_S) = +\sin[-(2\beta - 0)]. \quad (12)$$

Since Eq. (10) applies to excellent accuracy to the decay  $B_d \rightarrow J/\psi K_S$  – the point is that penguins enter essentially with the same weak phase as the leading tree contribution – it is usually referred to as the “gold-plated” mode to measure the angle  $\beta$  of the unitarity triangle<sup>11</sup>.

In the case of  $B_d \rightarrow \pi^+\pi^-$  mixing-induced CP violation would measure  $-\sin(2\alpha)$  in a clean way through

$$\mathcal{A}_{\text{CP}}^{\text{mix-ind}}(B_d \rightarrow \pi^+\pi^-) = -\sin[-(2\beta + 2\gamma)] = -\sin(2\alpha) \quad (13)$$

if there were no penguin contributions present. However, such contributions are there and destroy the *clean* relation Eq. (13). The corresponding uncertainties have been estimated recently<sup>12</sup> to be of the order 25%. Fortunately, as has been pointed out by Gronau and London<sup>13</sup>, the hadronic uncertainties arising from QCD penguin operators can be

eliminated by performing an isospin analysis involving in addition to  $B_d \rightarrow \pi^+\pi^-$  also the modes  $B_d \rightarrow \pi^0\pi^0$  and  $B^+ \rightarrow \pi^+\pi^0$ . Following this approach it is, however, not possible to control also the electroweak penguin contributions and the related uncertainties in a quantitative way<sup>14</sup>. As was found some time ago<sup>15</sup> (see also Ref.<sup>16</sup>), electroweak penguins may – in contrast to naïve expectations – play an important role in certain non-leptonic  $B$  decays, e.g. in  $B_s \rightarrow \pi^0\phi$ . This issue, which is due to the fact that the Wilson coefficient of one electroweak penguin operator increases strongly with the top-quark mass, led to considerable interest in the recent literature<sup>16,17,18</sup>. Detailed analyses<sup>17,18</sup> of the  $B \rightarrow \pi\pi$  isospin approach<sup>13</sup> to determine  $\alpha$  show, however, that electroweak penguins play there only a minor role. The corresponding uncertainty  $|\Delta\alpha|$  can be estimated<sup>19</sup> to be smaller than  $6^\circ$ . Consequently electroweak penguins are not expected to lead to serious problems in that approach. Since electroweak penguins enter in the case of  $B_d \rightarrow J/\psi K_S$  with the same weak phase as the leading tree contribution they do not affect the mixing-induced CP asymmetry measuring  $\sin(2\beta)$ .

Before we will focus on the  $B_s$  system in the next subsection, let me note one experimental problem of the Gronau–London approach<sup>13</sup>. It is related to the fact that it requires a measurement of  $\text{BR}(B_d \rightarrow \pi^0\pi^0)$  which is regarded as being very difficult. A recent analysis by Kramer and Palmer<sup>20</sup> indicates  $\text{BR}(B_d \rightarrow \pi^0\pi^0) \lesssim \mathcal{O}(10^{-6})$ . Therefore it is important to have also alternatives available to determine  $\alpha$  in a clean way. Such alternatives are already on the market. For example, Snyder and Quinn<sup>21</sup> have suggested to use  $B \rightarrow \rho\pi$  modes to accomplish this ambitious task. Another method<sup>22</sup> has been proposed recently by Buras and myself. It requires a simultaneous measurement of  $\mathcal{A}_{\text{CP}}^{\text{mix-ind}}(B_d \rightarrow \pi^+\pi^-)$  and  $\mathcal{A}_{\text{CP}}^{\text{mix-ind}}(B_d \rightarrow K^0\overline{K}^0)$  and determines  $\alpha$  with the help of the  $SU(3)$  flavor symmetry of strong interactions. Interestingly the penguin-induced decay  $B_d \rightarrow K^0\overline{K}^0$  may exhibit large CP-violating asymmetries<sup>23</sup> within the Standard Model due to interference effects between penguins with internal up- and charm-quark exchanges<sup>24</sup>. This feature was missed in several previous analyses.

Further papers dealing with the penguin uncertainties affecting the extraction of  $\alpha$  are given in Ref.<sup>25</sup>. Recently the decays  $B_d(t) \rightarrow \pi^+\pi^-$ ,  $B_d^0 \rightarrow \pi^-K^+$ ,  $B^+ \rightarrow \pi^+K^0$  and their CP-conjugates have been considered in Ref.<sup>26</sup> by using  $SU(3)$  flavor symmetry arguments. The corresponding observables may allow a determination of the angles  $\alpha$  and  $\gamma$  of the unitarity triangle.

## 2.2. The $B_s$ System in Light of $\Delta\Gamma_s$

The situation arising in the  $B_s$  system may be quite different from the  $B_d$  case because of the expected sizable width difference<sup>27</sup>  $\Delta\Gamma_s \equiv \Gamma_H^{(s)} - \Gamma_L^{(s)}$ . Here  $\Gamma_H^{(s)}$  and  $\Gamma_L^{(s)}$  denote the decay widths of the  $B_s$  mass eigenstates  $B_s^{\text{Heavy}}$  and  $B_s^{\text{Light}}$ , respectively. The major contributions to  $\Delta\Gamma_s$ , which may be as large as  $\mathcal{O}(20\%)$  of the average decay width, originate from  $\bar{b} \rightarrow \bar{c}c\bar{s}$  transitions into final states that are common both to  $B_s^0$  and  $\overline{B}_s^0$ . As has been pointed out by Dunietz<sup>28</sup>, due to this width difference already *untagged*  $B_s$  rates defined by

$$\Gamma[f(t)] \equiv \Gamma(B_s^0(t) \rightarrow f) + \Gamma(\overline{B}_s^0(t) \rightarrow f) \quad (14)$$

may provide valuable information about the phase structure of the observable

$$\xi_f^{(s)} = \exp\left(-i\Theta_{M_{12}}^{(s)}\right) \frac{A(\overline{B}_s^0 \rightarrow f)}{A(B_s^0 \rightarrow f)}, \quad (15)$$

where  $\Theta_{M_{12}}^{(s)}$  is the weak  $B_s^0 - \overline{B}_s^0$  mixing phase. This can be seen nicely by writing Eq. (14) in a more explicit way as follows:

$$\Gamma[f(t)] \propto \left[ \left( 1 + |\xi_f^{(s)}|^2 \right) \left( e^{-\Gamma_L^{(s)} t} + e^{-\Gamma_H^{(s)} t} \right) - 2 \operatorname{Re} \xi_f^{(s)} \left( e^{-\Gamma_L^{(s)} t} - e^{-\Gamma_H^{(s)} t} \right) \right]. \quad (16)$$

In this expression the rapid oscillatory  $\Delta M_s t$  terms, which show up in the *tagged* rates, cancel<sup>28</sup>. Therefore it depends only on the two exponents  $e^{-\Gamma_L^{(s)} t}$  and  $e^{-\Gamma_H^{(s)} t}$ , where  $\Gamma_L^{(s)}$  and  $\Gamma_H^{(s)}$  can be determined e.g. from the angular distributions<sup>29</sup> of the decay  $B_s \rightarrow J/\psi \phi$ .

To illustrate these untagged rates in more detail, let me discuss an estimate of the CKM angle  $\gamma$  using *untagged*  $B_s \rightarrow K^+ K^-$  and  $B_s \rightarrow K^0 \overline{K}^0$  decays that has been proposed very recently by Dunietz and myself<sup>30</sup>. Using the  $SU(2)$  isospin symmetry of strong interactions to relate the QCD penguin contributions to these decays (electroweak penguins are color-suppressed in these modes and thus play a minor role), we obtain

$$\Gamma[K^+ K^-(t)] \propto |P'|^2 \left[ (1 - 2|r| \cos \rho \cos \gamma + |r|^2 \cos^2 \gamma) e^{-\Gamma_L^{(s)} t} + |r|^2 \sin^2 \gamma e^{-\Gamma_H^{(s)} t} \right] \quad (17)$$

and

$$\Gamma[K^0 \overline{K}^0(t)] \propto |P'|^2 e^{-\Gamma_L^{(s)} t}, \quad (18)$$

where

$$r \equiv |r| e^{i\rho} = \frac{|T'|}{|P'|} e^{i(\delta_{T'} - \delta_{P'})}. \quad (19)$$

Here  $P'$  denotes<sup>31</sup> the  $\bar{b} \rightarrow \bar{s}$  QCD penguin amplitude,  $T'$  is the color-allowed  $\bar{b} \rightarrow \bar{u} u \bar{s}$  tree amplitude, and  $\delta_{P'}$  and  $\delta_{T'}$  are the corresponding CP-conserving strong phases. In order to determine  $\gamma$  from the untagged rates Eqs. (17) and (18) we need an additional input that is provided by the  $SU(3)$  flavor symmetry of strong interactions. If we neglect the color-suppressed current-current contributions to  $B^+ \rightarrow \pi^+ \pi^0$  we find<sup>31</sup>

$$|T'| \approx \lambda \frac{f_K}{f_\pi} \sqrt{2} |A(B^+ \rightarrow \pi^+ \pi^0)|, \quad (20)$$

where  $\lambda$  is the Wolfenstein parameter<sup>32</sup>,  $f_K$  and  $f_\pi$  are the  $K$  and  $\pi$  meson decay constants, respectively, and  $A(B^+ \rightarrow \pi^+ \pi^0)$  denotes the appropriately normalized  $B^+ \rightarrow \pi^+ \pi^0$  decay amplitude. Since  $|P'|$  is known from  $B_s \rightarrow K^0 \overline{K}^0$ , the quantity  $|r| = |T'|/|P'|$  can be estimated with the help of Eq. (20) and allows the extraction of  $\gamma$  from the part of Eq. (17) evolving with the exponent  $e^{-\Gamma_H^{(s)} t}$ .

Such an  $SU(3)$  flavor symmetry input to determine  $\gamma$  is not necessary in the case of the decays  $B_s \rightarrow K^{*+} K^{*-}$  and  $B_s \rightarrow K^{*0} \overline{K}^{*0}$ . The angular distributions of these decays provide much more information than the pseudoscalar-pseudoscalar modes and therefore the  $SU(2)$  isospin symmetry of strong interactions to relate the QCD penguin contributions suffices to extract<sup>30</sup> the CKM angle  $\gamma$ . The time-dependent angular distributions of the decays  $B_s \rightarrow D_s^{*+} D_s^{*-}$ ,  $B_s \rightarrow J/\psi \phi$  allow the determination<sup>30</sup> of the Wolfenstein parameter<sup>32</sup>  $\eta$ , and time-dependent studies of angular distributions for  $B_s$  decays caused by  $\bar{b} \rightarrow \bar{c} u \bar{s}$  quark-level transitions extract<sup>33</sup> *cleanly* and *model-independently* the CKM angle  $\gamma$ . To this end *untagged*  $B_s$  data samples are sufficient if  $\Delta \Gamma_s$  is sizable. Even if  $\Delta \Gamma_s$  should turn out to be too small for such an untagged analysis, once  $\Delta \Gamma_s \neq 0$  has been established experimentally, the formulae presented in Refs.<sup>30,33</sup> have to be used in order to determine the CKM phases correctly.

### 2.3. Triangle Relations

Let me discuss this class of strategies for extracting CKM phases by considering the charged  $B$  decays  $B^+ \rightarrow \overline{D}^0 K^+$ ,  $B^+ \rightarrow D^0 K^+$  and  $B^+ \rightarrow D_+^0 K^+$ , where  $|D_+^0\rangle = (|D^0\rangle + |\overline{D}^0\rangle)/\sqrt{2}$  denotes the CP-eigenstate of the neutral  $D$  system corresponding to eigenvalue  $+1$ . Using this CP-eigenstate it is an easy exercise to derive the amplitude relations

$$\sqrt{2}A(B^+ \rightarrow D_+^0 K^+) = A(B^+ \rightarrow D^0 K^+) + A(B^+ \rightarrow \overline{D}^0 K^+) \quad (21)$$

$$\sqrt{2}A(B^- \rightarrow D_+^0 K^-) = A(B^- \rightarrow \overline{D}^0 K^-) + A(B^- \rightarrow D^0 K^-) \quad (22)$$

which can be represented as two triangles in the complex plane. Taking into account moreover the relations

$$A(B^+ \rightarrow D^0 K^+) = A(B^- \rightarrow \overline{D}^0 K^-) e^{2i\gamma} \quad (23)$$

$$A(B^+ \rightarrow \overline{D}^0 K^+) = A(B^- \rightarrow D^0 K^-), \quad (24)$$

which arise from the fact that the corresponding transitions are pure tree decays, a *clean* determination of the CKM angle  $\gamma$  is possible by using the triangle relations Eqs. (21) and (22). This approach has been proposed by Gronau and Wyler<sup>34</sup>, a similar strategy using neutral  $B_d \rightarrow DK^*$  ( $D \in \{D^0, \overline{D}^0, D_+^0\}$ ) decays by Dunietz<sup>35</sup>. Since the corresponding triangles are expected to be very squashed ones because of certain color-suppression effects, and since one is furthermore dealing with the CP-eigenstate  $|D_+^0\rangle$ , this method is unfortunately expected to be very challenging from an experimental point of view.

In some sense the triangle construction discussed above represents a proto-type of triangle constructions which have been very popular over the recent two years. They have been presented in a series of interesting papers<sup>36</sup> by Gronau, Hernández, London and Rosner who have combined the  $SU(3)$  flavor symmetry of strong interactions with plausible dynamical assumptions to derive triangle relations among  $B \rightarrow \{\pi\pi, \pi K, K\overline{K}\}$  decay amplitudes. These relations should allow a determination of the weak phases of the CKM matrix and of strong final state interaction phases by measuring only the corresponding branching ratios which are all of the order  $10^{-5}$ . Although this approach is very attractive at first sight, it has certain limitations. In contrast to the Gronau–Wyler triangle relations<sup>34</sup>, the  $SU(3)$  triangle relations are not valid exactly but suffer from unknown non-factorizable  $SU(3)$ -breaking corrections<sup>31</sup>. Moreover QCD penguins with internal up- and charm-quark exchanges affect the extraction of the CKM angle  $\beta$  using these relations<sup>24</sup>. In addition also electroweak penguins<sup>15,16</sup> play an important role in some of these constructions. This issue is the subject of the following section.

### 3. The Role of Electroweak Penguins

While electroweak penguin effects are negligibly small in the CP asymmetry of  $B_d \rightarrow J/\psi K_S$  measuring  $\sin(2\beta)$  and are expected to be of minor importance in the  $SU(2)$  isospin construction to determine  $\sin(2\alpha)$  from  $B \rightarrow \pi\pi$  modes as I have noted already in Subsection 2.1, electroweak penguins spoil<sup>14</sup> the determination<sup>36</sup> of  $\gamma$  using  $B^+ \rightarrow \{\pi^0 K^+, \pi^+ K^0, \pi^+ \pi^0\}$   $SU(3)$  triangle relations. Several solutions have been proposed in the recent literature to solve this problem. For example, Gronau et al.<sup>17</sup> have proposed an

amplitude quadrangle for  $B \rightarrow \pi K$  decays that can be used in principle to determine  $\gamma$  irrespectively of the presence of electroweak penguins. Since one side of this quadrangle is related to  $\text{BR}(B_s \rightarrow \pi^0 \eta) = \mathcal{O}(10^{-7})$ , this approach is very difficult from an experimental point of view. Another  $SU(3)$ -based approach to extract  $\gamma$  using the charged  $B$  decays  $B^- \rightarrow \{\pi^- \bar{K}^0, \pi^0 K^-, \eta_8 K^-\}$  and  $B^- \rightarrow \pi^- \pi^0$ , where electroweak penguins are also eliminated, has been proposed by Deshpande and He<sup>37</sup>. Although this method should be more promising for experimentalists, it is affected by  $\eta$ - $\eta'$  mixing and other  $SU(3)$ -breaking effects and cannot be regarded as a clean measurement of  $\gamma$ .

In view of this situation it would be useful to determine the electroweak penguin contributions experimentally. The knowledge of the electroweak penguin amplitudes would allow several predictions, consistency checks and tests of certain Standard Model calculations<sup>19</sup>. As Buras and myself have shown in Ref.<sup>18</sup>, if the CKM angle  $\gamma$  is used as an input, the  $\bar{b} \rightarrow \bar{s}$  electroweak penguin amplitude can be determined from the branching ratios for  $B^+ \rightarrow \pi^+ K^0$ ,  $B^+ \rightarrow \pi^0 K^+$ ,  $B^- \rightarrow \pi^0 K^-$ ,  $B_d^0 \rightarrow \pi^- K^+$  and  $\bar{B}_d^0 \rightarrow \pi^+ K^-$  by using only the  $SU(2)$  isospin symmetry of strong interactions. Taking into account that electroweak penguins are dominated by internal top-quark exchanges and using the  $SU(3)$  flavor symmetry, the  $\bar{b} \rightarrow \bar{d}$  electroweak penguin amplitude can be determined from the  $\bar{b} \rightarrow \bar{s}$  amplitude<sup>18</sup>. The central input needed to accomplish this ambitious task – the CKM angle  $\gamma$  – can also be determined approximately with the help of the branching ratios for  $B^+ \rightarrow \pi^+ K^0$ ,  $B_d^0 \rightarrow \pi^- K^+$ ,  $\bar{B}_d^0 \rightarrow \pi^+ K^-$  and  $B^+ \rightarrow \pi^+ \pi^0$ . This approach<sup>19</sup> should be interesting for experiments with limited statistics. In this respect also the estimate<sup>30</sup> of  $\gamma$  using untagged  $B_s \rightarrow K^+ K^-$ ,  $B_s \rightarrow K^0 \bar{K}^0$  decays discussed in Subsection 2.2 may be helpful. Measuring in addition the branching ratios for  $B^+ \rightarrow \pi^0 K^+$  and  $B^- \rightarrow \pi^0 K^-$  the electroweak penguin amplitudes can be determined as discussed in Ref.<sup>18</sup>. Once the CKM angle  $\gamma$  can be fixed by using absolutely clean methods, e.g. the ones proposed in Refs.<sup>33,34,38</sup>, the determination of electroweak penguins could be improved considerably. That should lead to interesting and valuable insights into the world of electroweak penguins. An exciting future may lie ahead of us!

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